10/31 Recitation

1. Geometrically determine the eigenvalues and eigenvectors of the matrices associated to the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by:
   (a) the identity transformation ($T(\mathbf{v}) = \mathbf{v}$)

   (b) projection of vectors onto the line $y = x$.

   (c) the trivial transformation ($T(\mathbf{v}) = \mathbf{0}$) for all $\mathbf{v} \in \mathbb{R}^2$.

2. Let A be an invertible matrix and let λ be an eigenvalue of A. Show that $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.

3. The matrix $M = \begin{bmatrix} .95 & .1 & 0 \\ .05 & .9 & .5 \\ 0 & 0 & .5 \end{bmatrix}$ describes the population migration between three states: A, B, and C. We assume it is a closed system: nobody is born, nobody dies, and nobody moves in or out of these states. Note that the first column says that in any given year, of the people living in A, 95% of people stay in A, 5% of residents move to B, and 0% of residents move to C.

   Suppose 50% of the total population of these three states lives in A, 40% lives in B, and 10% lives in C. Let $\mathbf{x}_0$ be the associated initial state vector. In this problem, you will determine what happens to the populations in the long run. Complete "Worksheet Problem 1" in the **Julia11EigenvaluesEigenvectors** notebook.

4. Suppose the weather on a given day depends only on the weather the previous day. If the weather is sunny one day, then there is a 90% chance that it is sunny the next day and a 10% chance that it is rainy the next day. On the other hand if the weather is rainy, then there is a 50% chance that it is sunny the next day and a 50% chance that it is rainy the next day. Complete "Worksheet Problem 2" in the **Julia11EigenvaluesEigenvectors** notebook.

5. **Looking Ahead** After completing the above two problems, you will have observed something special about Markov chains. Those examples converged to an eigenvector corresponding to the eigenvalue 1 (this is called a *steady state vector*). We'll see why (and under which conditions) this happens next week.